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TURBULENT FLOW AND HEAT TRANSFER OF A CHEMICALLY REACTING GAS MIXTURE IN A CHANNEL BEHIND AN ACCELERATING PISTON

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This article examines the nonsteady turbulent motion of a recombining gas in a chamber behind an accelerating piston. The chamber is a section of a cylindrical tube bounded on the left by a stationary wall and on the right by the piston. The evacuated section of the channel is located to the right of the piston. Before beginning motion, the partly dissociated gas – at a fairly high pressure – is uniformly distributed over the entire volume of the chamber, while the position of the piston is fixed. The piston is released at a certain moment of time taken as the initial moment and begins to accelerate toward the free end of the tube under the pressure of the hot gas. Expansion of the region occupied by the gas and the exchange of heat with the relatively cold wall of the channel lead to intensive recombination in the flow.

Our goal here is to construct a mathematical model of the given process and to study its gasdynamic features and criterional relations to determine parameters of the dynamic and thermal effects of the flow on the channel wall.

To describe the gas flow in the present case, it is best to use the Reynolds equations in the "narrow channel" approximation [1]. Together with the energy equations for a two-component, chemically active mixture and the heat-conduction equation for the wall, these equations have the form

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(\rho v r)=0; \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{\Sigma}\frac{\partial u}{\partial r}\right),\quad \frac{\partial p}{\partial r}=0; \tag{2}$$

$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial r}\right)=\frac{dp}{dt}+\mu_{\Sigma}\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho a_{\Sigma}\frac{\partial h}{\partial r}+r\rho a\,(\mathrm{Le}-1)\,\Delta h\frac{\partial c}{\partial r}\right],\quad p=\rho\tilde{R}T\,(1+c)/(2M); \tag{3}$$

$$\rho_w c_w\frac{\partial T_w}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_w\frac{\partial T_w}{\partial r}\right),\quad \frac{dp}{dt}=\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x},\quad \mu_{\Sigma}=\mu+\mu_t,\quad a_{\Sigma}=a+a_t, \tag{4}$$

where t is time; x and r are cylindrical coordinates; u and v are components of the velocity vector; ρ, h, and p are the density, enthalpy, and pressure; μ and a are molecular viscosity and diffusivity; μ_t and a_t are the turbulence analogs of the transport coefficients; M and c

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are the molecular weight and the mass concentration of the dissociated component; Δh is the thermal effect of the reaction; $\tilde{R}$ is the universal gas constant; Le is the molecular Lewis number; ρ_w, c_w, and λ_w are the density, heat capacity, and thermal conductivity of the material of the wall; T_w is the temperature of the wall. All of the quantities in system (1)-(3) are averages (averaging of turbulent pulsations). Heat propagation in the channel wall is analyzed on the basis of the undimensional heat-conduction equation (4). The solution of this equation gives a good approximation of the temperature field, due to the brevity of the gasdynamic process.

It is expedient to use the molecular theory of inhomogeneous gases to calculate the properties of the medium. The only empirical aspect of this approach to finding the transport coefficients is determination of the forces of molecular interaction. The latter are needed to calculate the integral collisions. The behavior of interacting particles is described by the Lennard-Jones potential with numerical values of the "force constants" chosen in accordance with [2]. The collision integrals were approximated by means of the formulas in [3], while the viscosity coefficient of the two-component gas was calculated by the familiar relations in [4]. The thermal conductivity of the mixture was calculated using the formula obtained by Mason and Saxon [2] with allowance for the Aiken coefficient reflecting the polyatomic nature of the gases. The coefficient of binary diffusion - which enters into the molecular Lewis number - was determined by means of a formula corresponding to the first approximation in the Chapman-Enskog theory [5].

It was assumed that the chemical reactions were equilibrium reactions with respect to gasdynamic restructuring of the flow and the turbulent fluctuations. The concentrations of the components in the case of fast reactions were calculated using the equilibrium conditions $\bar{\dot{\omega}} = 0$ ($\dot{\omega}$ is mass velocity in the formation of the dissociated component, with the superimposed bar denoting averaging of the pulsating quantities). By following [6] and thus making it possible to account for the effect of temperature pulsation on the rate of formation of the component, we can represent the equilibrium condition in the form

$$\bar{\dot{\omega}} = [\dot{\omega}(T^+) + \dot{\omega}(T^-)]/2 = 0, \quad T^+ = T + \sqrt{\overline{T'^2}}, \quad T^- = T - \sqrt{\overline{T'^2}}. \tag{5}$$

We will calculate the standard deviation of the temperature pulsations using an equation for autocorrelations of enthalpy fluctuations and the relation $\overline{T'^2} = \overline{h'^2}/c^2_{peff}$, which is valid for fast reactions.

For the dissociation of diatomic gases we have [7]

$$\dot{\omega} = [K_f(1 - c) + 2K_b\rho c^2/M] \tag{6}$$

(K_f and K_b are the rate constants of the forward and reverse reactions). With allowance for (6), we write Eq. (5) as

$$(1 - c)(1 + s)/2 = c^2 [1/(K_n^+ T^+) + s/(K_n^- T^-)]\, p/(MR), \tag{7}$$

where $s = \exp[E^*/(\tilde{R}T^+) - E^*/(\tilde{R}T^-)]$; $K_n^+ = K_n(T^+)$; $K_n^- = K_n(T^-)$; $K_n = K_f/K_b$; E^* is the activation energy; R is the gas constant of the mixture. At $T' = 0$, Eq. (7) becomes the familiar Huldbert-Waage relation. The equilibrium constant K_n was calculated in accordance with the data in [8].

We determined the coefficients characterizing the molar transfer of momentum and heat by using a differential turbulence model based on equations proposed in [9] for eddy kinetic energy, an equation proposed in [10] for the turbulence scale, and equations for standard deviations of the fluctuations of the transverse component of velocity and enthalpy [11];

$$\rho\left(\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial x} + v\frac{\partial E}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu + b_1\mu_t)\frac{\partial E}{\partial r}\right] + \mu_t\left(\frac{\partial u}{\partial r}\right)^2 - b_2(\mu + b_1\mu_t)\frac{E}{L^2}; \tag{8}$$

$$\rho\left(\frac{\partial L}{\partial t} + u\frac{\partial L}{\partial x} + v\frac{\partial L}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu + b_3\mu_t)\frac{\partial L}{\partial r}\right] - b_4\frac{L}{E}\mu_t\left(\frac{\partial u}{\partial r}\right)^2 + Bb_6\sqrt{E}\rho\left(1 - \frac{L^2}{(R-r)^2}\right); \tag{9}$$

$$\rho\left(\frac{\partial \overline{h'^2}}{\partial t} + u\frac{\partial \overline{h'^2}}{\partial x} + v\frac{\partial \overline{h'^2}}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r\rho(a + b_7 a_t)\frac{\partial \overline{h'^2}}{\partial r}\right] + b_8\rho\sqrt{\overline{v'^2}}\sqrt{\overline{h'^2}}\frac{\partial h}{\partial r} - b_9\rho a\frac{\overline{h'^2}}{L_h^2} - b_{10}\rho\frac{\sqrt{E}}{L_h}\overline{h'^2}; \tag{10}$$

$$\rho\left(\frac{\partial \overline{v'^2}}{\partial t} + u\frac{\partial \overline{v'^2}}{\partial x} + v\frac{\partial \overline{v'^2}}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu + b_1\mu_t)\frac{\partial \overline{v'^2}}{\partial r}\right] + b_{11}\rho\frac{E^{3/2}}{L}\overline{v'^2} - b_{12}\mu\frac{\overline{v'^2}}{L^2} - 2\rho\overline{v'^2}\frac{\partial v}{\partial r} - \frac{\partial \mu}{\partial r}\frac{\partial \overline{v'^2}}{\partial r}. \tag{11}$$

Here, the values of the constants b_i ($i = \overline{1, 12}$) were chosen in accordance with the data in [9, 10] and as a result of optimization of the calculations in our analysis of the similarity distributions for the averaged turbulence characteristics. In this case, the coefficient expressing the turbulent transfer of momentum μ_t was determined from the relation [9, 12]

$$\mu_t/\mu = \alpha \operatorname{Re}_t \left[1 - \exp\left(-\sigma_2 \operatorname{Re}_t^2\right) + \sigma_3 \operatorname{Re}_t^{1/2} \exp\left(-\sigma_1 \operatorname{Re}_t^2\right)\right], \quad \operatorname{Re}_t = \rho \sqrt{\varphi \bar{E}} L/\mu, \tag{12}$$

where Φ is the fraction of energy of the turbulent moles responsible for exchange processes. We find this quantity as follows:

$$\varphi = 1 - \exp\left(-\frac{K}{K_0}\frac{t}{t_t}\right).$$

Here, K is a constant determined on the basis of the agreement with the empirical data in [13]; $t_t = L/\sqrt{E}$ is the time of the turbulent pulsations, calculated from local values of the turbulence characteristics; $K_0 = D/\ell_c^0$ is a parameter characterizing the transience of the process; ℓ_c^0 is the initial length of the chamber.

We will represent the turbulent flow of heat in the form [11]

$$\overline{v'h'} = b \sqrt{\overline{v'^2}} \sqrt{\overline{h'^2}}, \quad b = \text{const}. \tag{13}$$

Then in the case of the flow of a two-component medium and "extensive" equilibrium reactions (approximately with allowance for the assumption $\frac{\partial c}{\partial T} T' \gg \frac{\partial c}{\partial p} p'$ we have

$$a_t = \left(b \sqrt{\overline{v'^2}} \sqrt{\overline{h'^2}} + \frac{a}{2} (\mathrm{Le} - 1) \Delta c_p \frac{\partial \overline{c'^2}}{\partial r} \Big/ \frac{\partial c}{\partial T}\right) \Big/ \frac{\partial h}{\partial r}, \quad \overline{c'^2} = \left(\frac{\partial c}{\partial T}\right)^2 \overline{h'^2}/c_{p\mathrm{eff}}^2, \quad c_{p\mathrm{eff}} = c_p + \frac{\partial c}{\partial T} \Delta h. \tag{14}$$

It should be noted that the gradient representation for turbulent heat flow used in writing Eq. (3) is not essential in the solution of the thermal problem by means of Eq. (13).

The above-described system of equations was integrated with initial distributions corresponding to the state of rest and with the following boundary conditions: symmetry on the flow axis; adhesion for the mean and fluctuation velocities; continuity for the temperature fields at the impermeable boundaries. The numerical solution of the problem is constructed on the basis of efficient implicit finite-difference schemes. The method used to construct the solution was described in detail in [14, 15]. We first tested the algorithm in trivial and self-similar cases of flow and heat transfer. Some of the results of tests of the solutions of Eqs. (10) and (11) are shown in Figs. 1 and 2.

The data shown in Fig. 1 corresponds to flow calculations performed with $\mathrm{Re} = 4.235 \cdot 10^5$, $\mathrm{Tu}_0 = 5\%$, $L_0 = 0.125D$, where lines 1-4 are for x/D = 10, 100, 170, and 200 and the circles represent Laufer's measurements [16]. Figure 2 shows the distribution of dimensionless rms, fluctuations of enthalpy $\sigma_h = \sqrt{\overline{h'^2}}/(q_w/(\rho u_*))$ in relation to the universal transverse coordinate $y^+ = \rho y u_*/\mu$. The circles show the results from [17], while lines 1 and 2 correspond to x/D = 10, 200 ($\mathrm{Tu}_0 = 5\%$, $L_0 = 0.125D$). It is evident from the graphs that there is good agreement between the theoretical profiles and the similarity distributions obtained in the experiments in [16, 17].

Calculations of gas flows behind an accelerating piston with the following values for the initial parameters: $\ell_c^0 = 0.3$ m, $R = 3.95 \cdot 10^{-3}$ m, $R_w = 8 \cdot 10^{-3}$ m, Pr = 0.674, $T^0 = 5700$ K, $T_w^0 = 300$ K, $\lambda_w^0 = 46$ W/(m·K), $\gamma = 1.333$, $p^0 = 2 \cdot 10^8$ N/m^2, $\sigma = m_1/m_2 = 0.918$, $m_2 = 10^{-3}$ kg, $\mathrm{Tu}^0 = 10^{-4}$, $L^0 = 0.01 \cdot R$. Here, R_c and R are the external and internal radii of the channel; σ is the ratio of the mass of the gas to the mass of the piston; γ is the adiabatic exponent; Tu is turbulence intensity; a zero superscript denotes the initial moment of time. We used hydrogen as the heat carrier. Figure 3 shows distributions of the intergral characteristics of the flow: the pressure on the left boundary of the piston p_2 and the velocity of the piston u_2 (curves 1 and 2) in relation to its position as characterized by the coordinate $x_2 = x_2(t)$. The solid lines are for the recombining gas, while the dashed lines are for the "frozen" flow. It is evident that chemical changes can have a significant effect on the gasdynamic pattern of the flow. Thus, the recombining gas has a high pressure and creates conditions which tend to increase piston velocity. Velocity increases 21% on the section $\ell = 0.7$ m due to the conversion of chemical energy into thermal energy. The increase in pressure on the piston by the final moment of the process is 250%.

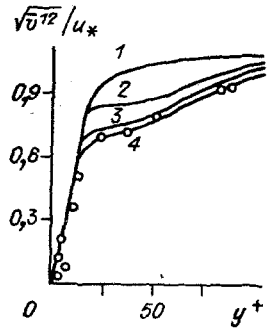

Fig. 1

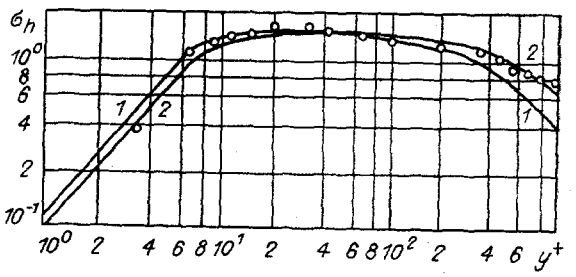

Fig. 2

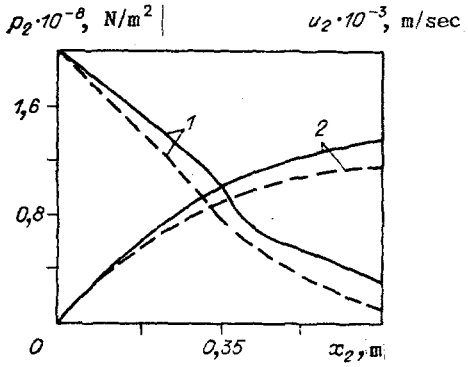

Fig. 3

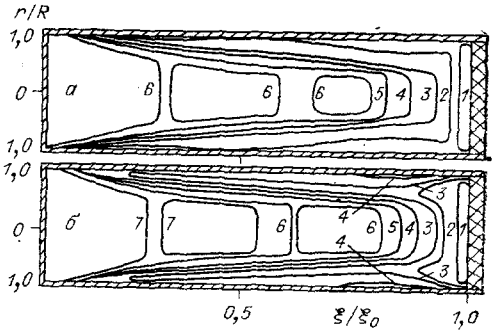

Fig. 4

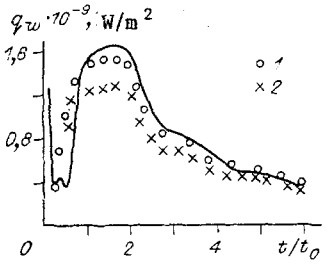

Fig. 5

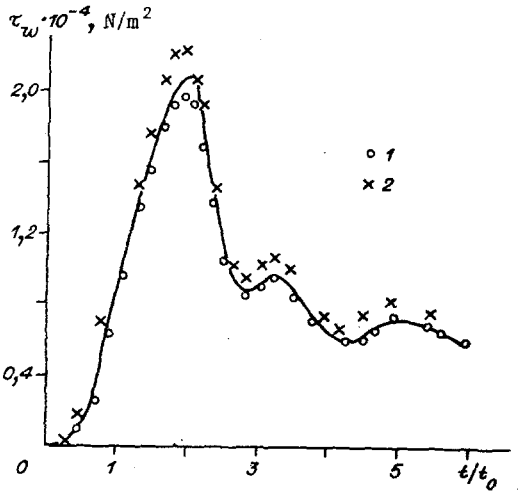

Fig. 6

Some of the results of calculation of the mean turbulence characteristics are shown in Fig. 4, where a are the rms fluctuations of the transverse component of velocity $\sqrt{v'^2}/u_*$ for the moment of time $t/t_0 = 3$. Curves 1-6 correspond to $\sqrt{v'^2}/u_* = 10, 2, 1, 0.6, 0.25, 0.05$; Fig. 4b shows rms fluctuations of enthalpy $\sqrt{h'^2}/h_*$ for the moment of time $t/t_0 = 3$. Lines 1-7 correspond to $\sqrt{h'^2}/h_* = 8.5, 4, 3, 2, 1, 0.2, 0.048$. It is evident (Fig. 4a) that the regions of high values of $\sqrt{v'^2}/u_*$ are concentrated in the area behind the piston and near the lateral surface of the channel. The main factor in the mechanism responsible for the generation of turbulence near the walls is the presence of large gradients of mean velocity in these regions. Due to the brevity of the process, the contribution of diffusion to the transfer of momentum from the walls to the flow core is limited, while the generation of turbulence within this region is impeded by high values of gas-particle acceleration and the absence of shear in the axial part of the channel. The results of our calculations show that the maximum of the quantity $\sqrt{v'^2}/u_*$ (in the zone where the intensity of the turbulent pulsations increases) in the channel section $\xi/\xi_0 = 0.5$ for the moment of time $t/t_0 = 3$ is approximately 3.5 times greater than the corresponding values with stationary motion of the gas on

the flow-stabilizing section. The zero subscripts here and below indicate characteristic values of the parameters ($t_0 = \ell_c^0/\sqrt{\gamma RT^0}$ is the time of the process and ξ_0 is the total mass of the gas). The character of distribution of the rate of fluctuation of enthalpy $\sigma_h = \sqrt{h'^2}/h_*$ (Fig. 4b) is relatively complex by comparison, which can be attributed to the interaction of the thermal and hydrodynamic fields. However, as for velocity, the regions characterized by a high enthalpy fluctuation rate are localized near the piston and the lateral surface of the channel.

The lines in Figs. 5 and 6 show results of calculations of the parameters that characterize the thermal and dynamic effects of a chemically reactive flow on the channel wall in accordance with the above-described model. Figure 5 shows the dependence of heat flux q_w on the dimensionless parameter t/t_0 on the channel section $x = -0.05$ m. We note that the motion of the piston begins from the position $x = 0$ at $t = 0$. Here and in Fig. 6, points 1 show results calculated with Eqs. (15)-(18), while Fig. 2 shows the same relations with $\tilde{c}_{peff}/c_p = 1$. It is evident that the distribution of q_w in the chemically reacting flow differs appreciably from the motion of an inert medium [15] in regard to the presence of characteristic "steps." The existence of the steps is due to an intensification of wave effects in the reacting flow as a result both of a decrease in sonic velocity and the possibility of the conversion of thermal energy into chemical as well as mechanical energy.

Despite its conservative nature, the friction coefficient in our test (Fig. 6) turned out to be more sensitive to chemical transformations. However, satisfactory agreement can be obtained between the results calculated from the above-described model and found with the criterional relations for the Nusselt number and the friction coefficient. To do this, expressions which are valid for the flow of an inert medium [15] are supplemented by the complexes which contain $\tilde{c}_{peff}/c_p$. Here, $\tilde{c}_{peff}$ is taken as the effective heat capacity averaged over the interval from T_w to $\bar{T}$, while c_p is the "frozen" heat capacity of the gas mixture. The calculations show that the following are good approximations for the case of the flow of a dissociating medium behind an accelerating piston

$$\zeta = 0{,}131\,\mathrm{Re}^{-0{,}18}\left(\frac{\Theta}{T_w}\right)^{0{,}215}\left(\frac{\tilde{c}_{peff}}{c_p}\right)^{-0{,}35}; \tag{15}$$

$$\mathrm{Nu} = 0{,}0162\mathrm{Re}^{0{,}82}\mathrm{Pr}^{0{,}4}\left(\frac{\Theta}{T_w}\right)^{0{,}215}\left(\frac{\tilde{c}_{p\,\mathrm{eff}}}{c_p}\right)^{0{,}45}; \tag{16}$$

$$\mathrm{Re} = D\bar{\rho}\bar{u}/\mu(\bar{T}),\ \ \mathrm{Pr} = c_p\mu(\bar{T})/\lambda(\bar{T}),\ \ \Theta = \bar{T} + r_w\bar{u}^2/(2c_p),\ \ r_w = \sqrt[3]{\mathrm{Pr}}; \tag{17}$$

$$\tau_w = (\zeta/8)\bar{\rho}\bar{u}^2,\ \ q_w = \mathrm{Nu}(\Theta - T_w)\lambda(\bar{T})/D. \tag{18}$$

Here, ζ is the friction coefficient; Nu and Pr are the Nusselt and Prandtl numbers; Θ is the flow stagnation temperature; r_w is the coefficient of restriction; $\bar{\rho}$ and $\bar{T}$ are the density of the gas averaged over the cross section and the mean-mass temperature of the flow; T_w is the temperature of the internal surface of the channel.

Thus, Eqs. (15)-(18) can be used to calculate friction and heat transfer both for a flow of inert gases and a flow of chemically reactive gases behind an accelerating piston in a channel.

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CASCADE PROCESSES AND FRACTALS IN TURBULENCE

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The idea of homogeneity is now giving way to the less restrictive idea of fractal self-similarity (see, for example, [1-4]). Cascade models, which have been used successfully for studying homogeneous turbulence, could also be useful for studying fractal turbulence. On the other hand, these cascade models can be improved by taking into account the fractal structure of real turbulence. A great deal of experimental material has now been accumulated. This material needs to be organized and interpreted. In the present paper we examine some aspects of cascade processes taking into account the fractal structure of turbulence. First, we establish a relation between the form of the energy spectral density in the scaling interval and the fractal dimension of the surface of the hydrodynamic fields. This relation is important, in particular, for atmospheric turbulence and is confirmed by direct observations of atmospheric hydrodynamic fields, performed by different authors. An analogous investigation was also performed for two-dimensional turbulence, the computational results for which are confirmed by comparing with oceanographic computational data. Second, a relation between the constant in the Kolmogorov–Obukhov spectral law and the intermittency coefficient is established by taking into account the fractal structure.

Suppose that when turbulence arises it has a patchy character, i.e., the nonturbulent region contains separate subregions occupied with turbulent fluid [5] (criteria for distinguishing between the subregions are given, for example, in [6]). Since the fluid particles in the turbulent liquid strive to move away from each other [7, 8], one would expect that in time these regions will expand on the average. Moreover, this property of fluid particles in a turbulent liquid should, in general, cause the turbulent part of the liquid to strive constantly to increase the total area of the boundary separating it from the nonturbulent fluid. Is this process unbounded or can it saturate? If a self-similar situation is established, then the total area of the surface separating the turbulent liquid from the nonturbulent liquid will approach infinity, and this surface will become a fractal with fractal dimension $D_\sigma > 2$ (in three-dimensional space).

We introduce the probability density $\rho(\ell)$ for encountering a turbulent subregion with characteristic size ℓ. By definition of the probability density the total area separating the turbulent and nonturbulent regions in the interval of self-similarity is given by

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